

# The Mass Spectrum of Neutrinos

E. Capelas de Oliveira,<sup>\*</sup> W. A. Rodrigues Jr.<sup>†</sup> and J. Vaz Jr.<sup>‡</sup>  
 Institute of Mathematics, Statistics and Scientific Computation  
 IMECC-UNICAMP  
 13083-859 Campinas, SP, Brazil

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## Abstract

In a previous paper we showed that Weyl equation possess superluminal solutions and moreover we showed that those solutions that are eigenstates of the parity operator seem to describe a coupled pair of a monopole anti-monopole system. This result suggests to look for a solution of Maxwell equation  $\partial F^\infty = -\mathbf{g}\mathcal{J}$  with a current  $\mathcal{J}$  as source and such that the Lorentz force on the current is null. We first identify a solution where  $\mathcal{J} = \gamma^5 J_m$  is a space-like field (even if  $F$  is not a superluminal solution of the homogeneous Maxwell equation). More surprisingly we find that there exists a solution  $F$  of the free Maxwell  $\partial F = 0$  that is equivalent to the non homogeneous equation for  $F^\infty$ . Once this result is proved it suggests by itself to look for more general subluminal and superluminal solutions  $\mathfrak{F}$  of the free Maxwell equation equivalent to a non homogeneous Maxwell equation for a field  $\mathfrak{F}_0$  with a current term as source which may be subluminal or superluminal. We exhibit one such subluminal solution, for which the Dirac-Hestenes spinor field  $\psi$  associated the electromagnetic field  $\mathfrak{F}_0$  satisfies a Dirac equation for a bradyonic neutrino under the *ansatz* that the current is  $ce^{\lambda\gamma^5}\mathbf{g}\psi\gamma^0\tilde{\psi}$ , with  $\mathbf{g}$  the quantum of magnetic charge and  $\lambda$  a constant to be determined in such a way that the auto-force be null. Together with Dirac's quantization condition this gives a quantized

mass spectrum (Eq.(49)) for the neutrinos, with the masses of the different flavor neutrinos being of the same order of magnitude (Eq.(50)) which is in accord with recent experimental findings. As a last surprise we show that the mass spectrum found in the previous case continues to hold if the current is taken spacelike, i.e.,  $ce^{\lambda\gamma^5}\mathbf{g}\psi_{>}\gamma^3\tilde{\psi}_{>}$  with  $\psi_{>}$ , in this case, satisfying a tachyonic Dirac-Hestenes equation.

## 1 Maxwell Equation and Neutrinos

In a previous note we recalled that Weyl equation satisfied by a Weyl spinor field that is an eigenvector of the parity operator seems to describe a couple monopole anti-monopole system (CMAMS) propagating together. If this is indeed the case than the coupled system must be solution of a Maxwell equation with a dipolar magnetic current and such that in order for the CMAMS to be stable the resulting Lorentz force on the CMAMS (at least classically) is null. We use the same notations and conventions as in [5, 4]. All phenomena is supposed to be well described in Minkowski spacetime containing a preferred inertial frame eliminating causal paradoxes and breaking active Lorentz invariance [4]. Then, the Maxwell equation describing the system we are interested is

$$\partial F^\infty = -\mathbf{g}\gamma^5 J_m = \mathbf{g} \star J_m \quad (1)$$

where  $\mathbf{g} \in \mathbb{R}$  denotes magnetic charge,  $F^\infty \in \sec \bigwedge^2 T^*M \hookrightarrow \sec \mathcal{C}\ell(M, g)$ ,  $J_m \in \sec \bigwedge^1 T^*M \in$

<sup>\*</sup>capelas@ime.unicamp.br

<sup>†</sup>walrod@ime.unicamp.br or walrod@mpc.com.br

<sup>‡</sup>vaz@ime.unicamp.br

$\text{sec } \mathcal{C}\ell(M, g)$ . Moreover,  $\partial$  is the Dirac operator acting on sections of the Clifford bundle of differential forms  $\mathcal{C}\ell(M, g)$  and  $\star$  stands for the Hodge star operator. Now, the Lorentz force acting on  $J_m$  is  $\mathbf{F} = J_m \lrcorner \star F^\infty$  and we want that

$$\mathbf{F} = J_m \lrcorner \star F^\infty = 0. \quad (2)$$

Let  $\{e_\mu = \partial/\partial x^\mu\}$  be an orthonormal basis for  $TM$  where  $(x^0, x^1, x^2, x^3) = (t, x, y, z)$  are coordinates in the Einstein-Lorentz-Poincaré gauge naturally adapted to the inertial frame  $e_0 = \frac{\partial}{\partial t}$ . Let  $\{\gamma^\mu\}$  be a basis for  $T^*M$  dual to the basis  $\{e_\mu\}$ . Write  $J_m = \rho_m \gamma^0 - j_{mi} \gamma^i$  and introduce engineering notation (using the well known fact that the even sub-bundle  $\mathcal{C}\ell^0(M, g)$  of  $\mathcal{C}\ell(M, g)$  is isomorphic to the Pauli bundle) writing

$$F^\infty = \frac{1}{2} F_{\mu\nu} \gamma^\mu \gamma^\nu := \mathbf{E}_\infty + \gamma^5 \mathbf{B}_\infty = \mathbf{E}_\infty - \mathbf{i} \mathbf{B}_\infty \quad (3)$$

with

$$\begin{aligned} \mathbf{E}_\infty &= \sum_{i=1}^3 F_{0i} \gamma^0 \gamma^i = \sum_{i=1}^3 F_{0i} \sigma_i, \\ \mathbf{B}_\infty &= \frac{1}{2} \sum_{i,j=1}^3 F_{ij} \gamma^i \gamma^j = -\frac{1}{2} \sum_{i,j=1}^3 F_{ij} \gamma \sigma_i \sigma_j \\ \star F^\infty &= \mathbf{i} F^\infty = \mathbf{B}_\infty + \mathbf{i} \mathbf{E}_\infty. \end{aligned} \quad (4)$$

Projecting  $\mathbf{F}$  in Eq.(2) in the Pauli bundle selected by the inertial reference frame  $e_0$  we get<sup>1</sup>

$$\mathbf{F} \gamma^0 = \mathbf{j}_m \bullet \mathbf{B}_\infty - (\rho_m \mathbf{B}_\infty + \mathbf{j}_m \times \mathbf{E}_\infty), \quad (5)$$

and we immediately realize that we can have  $\mathbf{F} = 0$  if  $J_m$  is a spacelike or a timelike current. For the first case it exists an inertial reference frame where the current is *transcendent*<sup>2</sup>, i.e., where  $\rho_m = 0$ ,  $\mathbf{j}_m \neq 0$ , but this is not, of course true in any reference frame. Thus to have  $\mathbf{F} = 0$  we need in the general case

$$\mathbf{j}_m = b \mathbf{E}_\infty, \quad \mathbf{E}_\infty \bullet \mathbf{B}_\infty = 0, \quad (6)$$

which force us to take  $\mathbf{B}_\infty = 0$ . This means that in Eq.(1) we must take

$$F^\infty = \sum_{i=1}^3 F_{0i} \gamma^0 \gamma^i = \sum_{i=1}^3 F_{0i} \sigma_i. \quad (7)$$

<sup>1</sup>The Euclidian scalar product will be denoted by the symbol “ $\bullet$ ”.

<sup>2</sup>We take  $e_\mu = \partial/\partial x^\mu = \partial/\partial t$  as that system and say that the CMAMS is transcendent.

We now write Eq.(1) in the Pauli algebra associated to the inertial frame for the case of a spacelike current and in the reference frame where it is transcendent. We have using that  $\rho_m = 0$  and  $b = 1$ ,

$$\begin{aligned} \mathbf{j}_m &= J_m \gamma_0 = j_{mi} \gamma_i \gamma_0 = b \mathbf{E}_\infty = b \sum_{i=1}^3 F_{0i} \sigma_i \\ &= b \sum_{i=1}^3 F_{0i} \gamma^0 \gamma^i, \end{aligned} \quad (8)$$

and thus

$$J_m = (\sum_{i=1}^3 F_{0i} \gamma^0 \gamma^i) \gamma^0 = F^\infty \gamma^0. \quad (9)$$

Then Eq.(1) becomes

$$\partial F^\infty = -\mathbf{g} \gamma^5 F^\infty \gamma^0. \quad (10)$$

It is quite clear that in the reference frame where the current is transcendent its support for each instant of time is  $\mathbb{R}^3$  and in that frame we suppose that  $F^\infty$  is static, i.e., it is not a function of  $t$ . Under these conditions we can write Eq.(10) as

$$-\gamma^i \gamma^0 \partial_i \sum_{i=1}^3 F_{0i} \sigma_i = \mathbf{g} \mathbf{i} \sum_{i=1}^3 F_{0i} \sigma_i \quad (11)$$

or yet,

$$\nabla \mathbf{E}_\infty = \mathbf{g} \mathbf{i} \mathbf{E}_\infty. \quad (12)$$

Finally observing that the total electric charge density involved in our problem is always null we must have  $\nabla \bullet \mathbf{E}_\infty = 0$  and thus Eq.(12) can be written

$$-\mathbf{i}(\nabla \wedge \mathbf{E}_\infty) = 2\mathbf{g} \mathbf{E}_\infty. \quad (13)$$

Taking into account that  $-\mathbf{i}(\nabla \wedge \mathbf{E}_\infty) := \nabla \times \mathbf{E}_\infty$  we end with the surprising result that the CMAMS electromagnetic field satisfies a force free equation, i.e.,

$$\nabla \times \mathbf{E}_\infty = 2\mathbf{g} \mathbf{E}_\infty. \quad (14)$$

Applying the  $\nabla \times$  to both members of that equation we get taking into account that  $\nabla \bullet \mathbf{E}_\infty = 0$  that

$$\nabla^2 \mathbf{E}_\infty + \mathbf{g}^2 \mathbf{E}_\infty = 0. \quad (15)$$

If we suppose that the possible values of the magnetic charge obeys Dirac quantization condition, i.e.  $e\mathbf{g} = n/2$ ,  $n \in \mathbb{Z}$ , then it results that the possible values of  $\mathbf{g}^2$  is also quantized.

Our surprises did not end yet. Indeed, we now show that a *free* electromagnetic field can simulate Eq.(1). In order to show this result write

$$\begin{aligned} F &= F^\infty \exp(\gamma^5 m \mathbf{t}) \\ &= \mathbf{E}_\infty \cos m \mathbf{t} - \mathbf{i} \mathbf{E}_\infty \sin m \mathbf{t} \\ &= \mathbf{E} + \mathbf{i} \mathbf{B}. \end{aligned} \quad (16)$$

Recalling again that the density of magnetic charge for a transcendent CMAMS is null we immediately realize that

$$\begin{aligned} \nabla \bullet \mathbf{E} &= \nabla \bullet \mathbf{B} = 0, \\ \nabla \times \mathbf{E} + \frac{\partial}{\partial \mathbf{t}} \mathbf{B} &= 0, \quad \nabla \times \mathbf{B} - \frac{\partial}{\partial \mathbf{t}} \mathbf{E} = 0 \end{aligned}$$

i.e.,  $\partial F = 0$ .

Then, taking into account that  $F^\infty$  is independent of time we can write

$$\partial(F^\infty \cos m \mathbf{t}) = +\gamma^5 \partial(F^\infty \sin m \mathbf{t}), \quad (17)$$

getting

$$\gamma^i \partial_i F^\infty = m \gamma^5 \gamma^0 F^\infty, \quad (18)$$

and since  $\gamma^0 \wedge F^\infty = 0$  we have that  $\gamma^0 F^\infty = \gamma^0 \lrcorner F^\infty + \gamma^0 \wedge F^\infty = -F^\infty \lrcorner \gamma^0 = -F^\infty \gamma^0$  and Eq.(18) becomes

$$\partial F^\infty = -m \gamma^5 F^\infty \gamma^0, \quad (19a)$$

$$\partial^2 F^\infty - m^2 F^\infty = 0. \quad (19b)$$

Eq.(19a) is exactly Eq.(1), the Maxwell equation supposed to describe a CMAMS with a *spacelike current* (by Eq.(19a)) once we take into account (in appropriate units)  $m$  to be proportional to  $\mathbf{g}$ , i.e.,

$$m = \mathbf{g}. \quad (20)$$

**Remark 1** *In what follows we will see that in our theory the proportional factor  $\mathbf{g}$  can take only a discrete set of values thus determining a discrete set of neutrinos mass values.*

**Remark 2** *Another very interesting fact is the following one. Let  $\{e_\mu = \partial/\partial x^\mu\}$  be a basis for*

*TM where  $(x^0, x^1, x^2, x^3) = (t, x, y, z)$  are coordinates in the Einstein-Lorentz-Poincaré gauge naturally adapted to the inertial frame  $e_0 = \partial/\partial x^0$  moving with respect to  $\mathbf{e}_0$ , i.e.,*

$$e_0 = \frac{1}{\sqrt{1-V^2}} e_0 + \frac{V}{\sqrt{1-V^2}} e_3. \quad (21)$$

*In that frame we have that*

$$\begin{aligned} F &= F^\infty \exp[\gamma^5 (\frac{mt}{\sqrt{1-V^2}} - \frac{mVz}{\sqrt{1-V^2}})] \\ &= F^\infty \exp[\gamma^5 (\omega t - kz)], \end{aligned} \quad (22)$$

*and of course  $\omega^2 - k^2 = m^2$ , i.e., we have a subluminal free electromagnetic field configuration. But, of course the magnetic current associated to  $F^\infty$  continues to be superluminal.*

This result suggests us to look in an arbitrary basis<sup>3</sup>  $\{e_\mu = \partial/\partial x^\mu\}$  of *TM* (where  $(x^0, x^1, x^2, x^3) = (t, x, y, z)$  are coordinates in the Einstein-Lorentz-Poincaré gauge naturally adapted to the inertial frame  $e_0 = \partial/\partial x^0$ ) for solutions of the free Maxwell equation

$$\partial \mathfrak{F} = 0, \quad (23)$$

describing subluminal and superluminal neutrinos, i.e., such that Eq.(23) be equivalent to another non-homogeneous Maxwell equation for  $\mathfrak{F}_0$  with a *subluminal or superluminal magnetic current source*, but in both cases the Lorentz force being  $J_m \lrcorner \mathfrak{F}_0 = 0$ .

To simplify our task we suppose that the solution we are looking for is propagating in the  $z$ -direction of the inertial frame  $e_0$  at speed  $v$  associated with the phase of the duality rotation. By this statement we means that our solution must have the form

$$\mathfrak{F} = \mathfrak{F}_0 \exp[-\gamma^5 (\omega t - kz)] = \mathfrak{F}_0 e^{-\gamma^5 \chi} \quad (24)$$

with  $\kappa = \omega \gamma^0 - k \gamma^3$  and  $\chi = \omega t - kz$ . We have two cases that need to be analyzed: (i)  $\kappa^2_{<} = \omega^2_{<} - k^2_{<} = m^2_{<}$  and (ii)  $\kappa^2_{>} = \omega^2_{>} - k^2_{>} = -m^2_{>}$ . For both cases (i) and (ii) we write respectively<sup>4</sup>

$$\mathfrak{F}^{<} = \mathfrak{F}_0^{>} e^{-\gamma^5 \chi_{<}}, \quad \mathfrak{F}^{>} = \mathfrak{F}_0^{<} e^{-\gamma^5 \chi_{>}}. \quad (25)$$

<sup>3</sup>The dual basis of  $\{e_\mu\}$  is denoted in what follows by  $\{\gamma^\mu\}$ .

<sup>4</sup>The notation even if confused at a first sight will becomes obvious in a while.

The equivalent Maxwell equation for  $\mathfrak{F}_0^>$  ( $\mathfrak{F}_0^<$ ) has a superluminal (subluminal) magnetic current  $\kappa_{<}\mathfrak{F}_0^> = \kappa_{>}\mathfrak{F}_0^<$  ( $\kappa_{>}\mathfrak{F}_0^< = \kappa_{<}\mathfrak{F}_0^>$ ). Indeed, for  $\mathfrak{F}_0^>$  we have from Eq.(24) that

$$\partial\mathfrak{F}_0^> = -\gamma^5\kappa_{<}\mathfrak{F}_0^>, \quad (26)$$

which means that  $\mathfrak{F}_0^>$  must also satisfy the *tachyonic* Klein-Gordon equation,

$$\partial^2\mathfrak{F}_0^> - m_{<}^2\mathfrak{F}_0^> = 0. \quad (27)$$

For the case (ii) the equivalent equation for  $\mathfrak{F}_0^<$  is again

$$\partial\mathfrak{F}_0^< = -\gamma^5\kappa_{>}\mathfrak{F}_0^< \quad (28)$$

but now it has a subluminal magnetic current  $\kappa_{>}\mathfrak{F}_0^< = \kappa_{>}\mathfrak{F}_0^<$  since Eq.(28) gives a *bradyonic* Klein-Gordon equation, i.e.,

$$\partial^2\mathfrak{F}_0^< + m_{>}^2\mathfrak{F}_0^< = 0. \quad (29)$$

To obtain a solution  $\mathfrak{F}_0^<$  satisfying Eqs.(28) and (29), we introduce a *Hertz potential*  $\Pi^> \in \sec \wedge^2 T^*M \in \sec \mathcal{C}\ell(M, g)$  satisfying  $\square\Pi^> = 0$  such that the electromagnetic potential is  $A^> = -\delta\Pi^>$ . Thus since  $\delta A^> = 0$  and  $\mathfrak{F} = dA^>$  it follows that  $\partial\mathfrak{F}^> = 0$ . The Hertz potential satisfies  $\square\Pi^> = 0$  and we look for solutions of the form

$$\Pi^> = \Phi_{>}(x, y)e^{\gamma^5(\omega_{>}t - k_{>}z)}\mathbf{B}, \quad (30)$$

where  $\mathbf{B}$  is a constant 2-form. It follows that

$$\begin{aligned} & -\omega_{>}^2\Phi_{>}(x, y)\exp[\gamma^5(\omega_{>}t - k_{>}z)]\mathbf{B} \\ & -\nabla_2^2\Phi_{>}(x, y)\exp[\gamma^5(\omega_{>}t - k_{>}z)]\mathbf{B} \\ & +k_{>}^2\Phi_{>}(x, y)\exp[\gamma^5(\omega_{>}t - k_{>}z)]\mathbf{B} = 0. \end{aligned} \quad (31)$$

Then, if we choose

$$\nabla_2^2\Phi_{>}(x, y) = m_{>}^2\Phi_{>}(x, y) \quad (32)$$

where  $\nabla_2^2$  means the 2-dimensional Laplacian we get the desired dispersion relation, i.e.,

$$\omega_{>}^2 - k_{>}^2 = -m_{>}^2, \quad (33)$$

and  $\mathfrak{F}_0^<$  satisfies Eq.(29), a bradyonic Klein-Gordon equation.

On the other hand to obtain a solution  $\mathfrak{F}_0^>$  satisfying Eq.(27), we introduce a *Hertz potential*  $\Pi^< = \Phi_{<}(x, y)e^{\gamma^5(\omega_{>}t - k_{>}z)}\mathbf{B}$ , repeat the above steps but impose that  $\nabla_2^2\Phi_{<}(x, y) = -m_{<}^2\Phi_{<}(x, y)$ .

Thus, we have proved as stated above that the free Maxwell equation has solutions with a bradyonic dispersion relation, which is equivalent to another Maxwell equation with a tachyonic magnetic current and also has a solution with tachyonic dispersion relation which is equivalent to another Maxwell equation with a bradyonic magnetic current. We now analyze the case of the bradyonic current associated to a coupled monopole anti-monopole pair (i.e., the subluminal neutrino). The analysis for the other case is similar, and conduces to the same mass spectrum.

## 2 Neutrino Mass Spectrum

It remains to answer a crucial question. What kind of spinor field is associated with a subluminal ( or a superluminal) field  $\mathfrak{F}_0^>$ ?

The answer is that since  $\mathfrak{F}_0^2 \neq 0$ , we must associated to  $\mathfrak{F}_0$  a Dirac-Hestenes spinor field [2] whose representative in the spinorial frame associated to  $e_0$  is  $\psi \in \sec \mathcal{C}\ell^0(M, g)$ . Moreover, we assume that  $\psi\tilde{\psi} \neq 0$ , which means that  $\psi$  has the following decomposition

$$\psi = \varrho^{1/2}e^{\beta\gamma^5/2}R, \quad (34)$$

where  $\rho$  and  $\beta$  are scalar functions and for each space-time point  $x$ ,  $R(x) \in Spin_{1,3}^e$  since we want to write for the field of each neutrino flavor<sup>6</sup> in Gaussian units

$$\mathfrak{F}_0 = \frac{2\pi e\hbar}{mc}\psi\gamma^2\gamma^1\tilde{\psi}, \quad (35)$$

where  $e$  is the electronic charge and  $m$  is a mass parameter to be determined experimentally. We return to Eq.(26) which we write from now on in Gaussian units as

$$\partial\mathfrak{F}_0 = \frac{4\pi}{c}\mathcal{J}. \quad (36)$$

<sup>5</sup>To simplify notation in this section  $\mathfrak{F}_0^<$  is denoted  $\mathfrak{F}_0$ .

<sup>6</sup>That it is always possible to write  $\mathfrak{F}_0$  as in Eq.(35) is proved in [8].

Under the above conditions we can show [8, 6] that Eq.(36) is equivalent (once we impose a very reasonable constraint to match the number of degrees of freedom  $\mathfrak{F}_0$  with the ones of  $\psi$ ) to the following Dirac-like equation,

$$\begin{aligned} \partial\psi\gamma^2\gamma^1 &= \frac{\Lambda c}{\hbar}\psi\gamma_0 e^{\beta\gamma^5} \\ &+ \gamma^5 \frac{Kc}{\hbar}\psi\gamma_0 e^{\beta\gamma^5} + \frac{e^{\beta\gamma^5}}{\rho} \left( \frac{\mathfrak{m}}{e\hbar} \right) \mathcal{J}\psi, \end{aligned} \quad (37)$$

where the scalar functions  $\Lambda$  and  $K$  are given by

$$\Lambda = \Omega \cdot S, \quad K = \Omega \cdot (\gamma^5 S), \quad (38)$$

with  $S = \frac{1}{2}R\gamma^2\gamma^1\tilde{R}$ ,  $\Omega = v^\mu\Omega_\mu$ ,  $v^\mu = (R\gamma^0\tilde{R}) \cdot \gamma^\mu$  and

$$\Omega_\mu = 2(\partial_\mu R)\tilde{R}. \quad (39)$$

In [6] by taking  $K = 0$  and  $\Lambda = \mathfrak{m}$  (the mass of the electron),  $J_m = c\mathfrak{g}\phi\gamma^0\tilde{\phi}$  (with  $\phi = e^{-\beta\gamma^5}\psi$ ) and considering the muon and the tau as excited states of electron (containing pairs of monopole anti-monopole), we determine the spectrum of the lepton family (giving the electron mass) using Dirac's quantization condition with  $e = e/3$  as minimum charge ( $e$  being the electronic charge). We found the values 105.5 MeV and 1785 MeV for the muon and tau masses, respectively.

Here instead we take  $\Lambda = 0$  and determine  $K$  by imposing the *ansatz*

$$\mathcal{J} = e^{\lambda\gamma^5} c\mathfrak{g}\psi\gamma^0\tilde{\psi}, \quad (40)$$

where  $\lambda$  is to be determined in such a way that the auto-force  $\mathcal{J} \lrcorner \mathfrak{F}_0 = 0$ . At once we find that this implies

$$\lambda = \beta \pm n\pi \quad (41)$$

and without loosing generality we take in what follows  $\lambda = \beta$ .

Then, after some algebra we get

$$\partial\psi\gamma^2\gamma^1 = \frac{m_1 c}{\hbar}\psi\gamma_0 + \gamma^5 \frac{m_2 c}{\hbar}\psi\gamma_0 \quad (42)$$

where

$$m_1 = \left( K \sin \beta + \frac{\mathfrak{m}g}{e} \cos \beta \right), \quad (43a)$$

$$m_2 = \left( K \cos \beta + \frac{\mathfrak{m}g}{e} \sin \beta \right). \quad (43b)$$

We now impose that  $m_1 = 0$ , which implies that

$$\tan \beta = -\frac{1}{K} \left( \frac{\mathfrak{m}g}{e} \right). \quad (44)$$

Then

$$m_2 = \frac{K^2 - \left( \frac{\mathfrak{m}g}{e} \right)^2}{\sqrt{K^2 + \left( \frac{\mathfrak{m}g}{e} \right)^2}}. \quad (45)$$

We take again the value  $e = e/3$ , and use Dirac's quantization condition ( $\alpha$  is the fine structure constant)

$$ge/3\hbar c = \frac{n}{2}. \quad (46)$$

With the use of Eq.(46), the Eq.(45) becomes ( $m_{\nu_n} = m_2$ )

$$m_{\nu_n} = \frac{K^2 - \frac{9\mathfrak{m}^2}{4\alpha^2}n^2}{\sqrt{K^2 + \frac{9\mathfrak{m}^2}{4\alpha^2}n^2}} \quad (47)$$

Now, if the neutrino are massive particles then the allowed values of  $n \in \mathbb{Z}$  in Eq.(47) must satisfy  $K \geq 3\mathfrak{m}n/2\alpha$  and we write

$$K = 3\mathfrak{m}N/2\alpha, \quad (48)$$

for some constant  $N$ , which does not need to be an integer. Then the spectrum is

$$\boxed{m_{\nu_n} = \frac{3\mathfrak{m}}{2\alpha} \frac{(N^2 - n^2)}{\sqrt{N^2 + n^2}}} \quad (49)$$

So, our formula for the spectrum depends on two parameters  $\mathfrak{m}$  and  $N$ , but what is really important to emphasize here is that the allowed masses for the possible neutrino mass flavors are all very near, which seems to be in agreement with recent experimental data [7]. Since there exists three different flavors we take provisorily, in order to get an estimative of the masses  $N = 3$ . Under the constraint given in [7] that the masses of the neutrinos is less than 0.28 eV we get  $\mathfrak{m} = 1.97 \cdot 10^{-4}$  eV and the neutrino masses results

$$m_{\nu_0} = 0.12 \text{ eV}, \quad m_{\nu_1} = 0.10 \text{ eV}, \dots, m_{\nu_2} = 0.056 \text{ eV}. \quad (50)$$

**Remark 3** We observe that with the values given in Eq.(50) we have  $m_{\nu_0}^2 - m_{\nu_1}^2 = 4.4 \cdot 10^{-5}$  and

$m_{\nu_1}^2 - m_{\nu_2}^2 = 6.86 \cdot 10^{-3}$ ,  $m_{\nu_0}^2 - m_{\nu_2}^2 = 16.46 \cdot 10^{-3}$  which although not equal to published data [3] are of the same order of magnitude, but more cannot be inferred here because the mass formula is very sensitive to the values of the parameters.

Now, returning to Eq.(42) we get

$$\partial\psi\gamma^2\gamma^1 = \gamma^5 \frac{m_{\nu_n}c}{\hbar} \psi\gamma_0. \quad (51)$$

This is a Dirac equation for a bradyonic neutrino since the following Klein-Gordon equation holds ( $\psi = \psi_<$ )

$$\partial^2\psi_< + \left(\frac{m_{\nu_n}c}{\hbar}\right)^2 \psi_< = 0. \quad (52)$$

**Remark 4** Note that if neutrinos are tachyonic particles satisfying

$$\partial^2\psi_> - \left(\frac{m_{\nu_n}c}{\hbar}\right)^2 \psi_> = 0, \quad (53)$$

we may find a mass spectrum identical to the one given by Eq.(49) following steps analogous to the ones that lead to Eq.(42). We choose

$$\mathfrak{F}_0^> = \frac{2\pi e\hbar}{mc} \psi_> \gamma^0 \gamma^3 \tilde{\psi}_>,$$

use  $m_1 = 0$ , but now take  $\mathcal{J} = ce^{\lambda\gamma^5} \mathbf{g}\psi_> \gamma^3 \tilde{\psi}_>$  (a superluminal current). We arrive to the following corresponding tachyonic Dirac-Hestenes equation for  $\psi_>$ :

$$\partial\psi_> = \gamma^5 \frac{m_{\nu_n}c}{\hbar} \psi_> \gamma^0. \quad (54)$$

Thus  $\psi_>$  satisfies Eq.(53).

We observe that the mass spectrum found for the neutrinos is compatible (with reasonable choice of the parameters of the theory) with the recent experimental results [7] showing that the sum of the masses of the known flavor neutrinos is less than 0.28 eV (95% CL), for Eq.(47) show that the masses for the different flavours do not differ very much, the lightest neutrino having a mass less than 0.056 eV. However under these conditions our mass spectrum is not compatible with the data of the OPERA experiment that (if reliable) gives a tachyonic muonic neutrino mass

of 120 MeV. Finally, it is necessary to mention that the idea that monopoles are excited states of the neutrino was first proposed by Lochak [1], but he never obtained results as the ones described above.

## References

- [1] Lochak, G., Wave Equation for a Magnetic Monopole, *Int. J. Theor. Phys.* **24**, 1019-1050 (1985).
- [2] Mosna, R. A., and Rodrigues, W. A. Jr., The Bundles of Algebraic and Dirac-Hestenes Spinor Fields, *J. Math. Phys.* **45**, 2945-2966 (2004). [arXiv:math-ph/0212033v5]
- [3] Nakamura, K., et al. (Particle Data Group), *J. Phys. G* **37**, 075021 (2010)
- [4] de Oliveira, E. Capelas, Rodrigues, W. A. Jr., and Vaz, J. Jr., Superluminal Neutrinos from OPERA Experiment and Weyl Equation. [arXiv:1110.2219v1 [math-ph]]
- [5] Rodrigues, W. A. Jr., and de Oliveira, E. Capelas, *The Many Faces of Maxwell, Dirac and Einstein Equations. A Clifford Bundle Approach*. Lecture Notes in Physics 722, Springer, Heidelberg, 2007.
- [6] Rodrigues, W. A. Jr., and Vaz, J. Jr., From Electromagnetism to Relativistic Quantum Mechanics, *Found. Phys.* **28**, 789-814 (1998).
- [7] Thomas, S. A. , Abdalla, F. B. , and Lahav, O., Upper Bound of 0.28 eV on Neutrino Masses from the Largest Photometric Redshift Survey, *Phys. Rev. Lett.* **105**, 031301 (2010).
- [8] Vaz, J. Jr., and Rodrigues, W. A. Jr., Equivalence of Dirac and Maxwell Equations and Quantum Mechanics, *Int. J. Theor. Phys.* **32**, 945-955 (1993).